Assessment of Crossover Schemes in Genetic Algorithms Applied for the Optimization of Photonic Crystals Band Gap

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Resumen: *Este trabajo consiste en comparar el desempeño de dos esquemas de cruzamiento nos algoritmos genéticos cando son aplicados para optimizar la banda prohibida fotónica de estructuras cristalinas fotónicas en dos dimensiones y con arreglos cuadrados y triangulares. Fue considerada a banda prohibida entre los dos primeros modos transversales magnéticos denominada TM_{12}. Las propiedades de los cristales fue calculada por medio de los elementos finitos en el dominio de la frecuencia.*

Abstract: *The aim of this work is the assessment of two crossover schemes in genetic algorithms applied for the optimization of the band gap of photonic crystal structures with two dimensional periodicity and considering the square and triangular lattices. It was considered the band gap between the first and the second transverse magnetic modes denoted TM_{12}. The propagation properties of the crystals are obtained by an efficient finite element method in the frequency domain.*

Palabras clave: Photonic Crystals Band Gap, Genetic Algorithm, uniform crossover.

1. Introduction

Optical devices based on photonic crystals are of great applicability in optical communication systems such as demultiplexers, resonant cavities, optical filters [Joannopoulos, 2008]. Photonic crystals exhibit forbidden band gaps where light cannot propagate at all in any direction. This singular property is explored for designing new optical devices. In order to optimize their behavior, the forbidden band gap can be maximized for specific modes of propagation, allowing wider bandwidths of operation. Some techniques for the optimization or design of photonic devices have been already published, the genetic algorithm applied for the maximization of photonic band gap [Malheiros 2007], the artificial immune system applied for the design of wavelength division multiplexers [Silva-Santos, 2010a] and for the design of optimal power splitters [Silva-Santos, 2010b].

In this work the forbidden band gap of two dimensional photonic crystals in square and triangular lattices is optimized, and it is maximized for the two first transverse magnetic modes denoted as TM_{12}, by using genetic algorithms. Results obtained by using the uniform crossover are compared with the ones previously published in [Malheiros 2007], where the two-points crossover has been adopted. It is expected that the two-point crossover scheme enhances the diversity of the population and consequently the search space, allowing in this way, finding other optimal solutions.

In all the simulations, 20 and 50 individuals are considered in each generation and the evolution happens for a thousand generations for each kind of crossover in order to assess their performance.

2. Two Dimensional Photonic Crystals

In a photonic crystal, the light is scattered by the periodical geometry being refracted and partially reflected at the materials boundaries. The result of the overlapping of the waves is called the mode and it exhibits a periodical pattern over the crystal. The mode depends on the operating wavelength, the refractive indexes of the composing materials, the size and the organization of the structure. In some specific band of frequencies, light cannot propagate in the crystal and these frequencies are called the forbidden band gap.

Because of the periodicity of the crystals, the analysis can be carried out only over a unitary cell by imposing periodical boundary conditions of Bloch type. For a unitary cell, there is a reciprocal lattice called the Brillouin region, where the wavenumber vectors which determine the propagation properties are well defined. See Figure 1.

The shadow regions in Figure 1.b and Figure 1.e are the irreducible Brillouin regions limited by the points $\Gamma=(0,0)\pi/a$, $X=(1,0)\pi/a$, and $M=(1,1)\pi/a$ for the square lattice and $\Gamma=(0,0)\pi/a$, $K=(2/3, 2\sqrt{3}/3)\ \pi/a$, and $M=(0, 2\sqrt{3}/3)\ \pi/a$ for the triangular lattice. The wavenumber vectors are shown in Figures 1.c and 1.f, it is clear that no propagation can occur for TM modes between the normalized frequencies from 0.2 to 0.24 and from 0.2 to 0.27, respectively.

For the simulations, the geometries are generated by using the software GiD, and the band structure is computed by using an efficient frequency domain finite element method (FEM) [Malheiros, 2011]. In a FEM formulation, the geometry is divided in triangles and the Maxwell Equations are solved numerically.

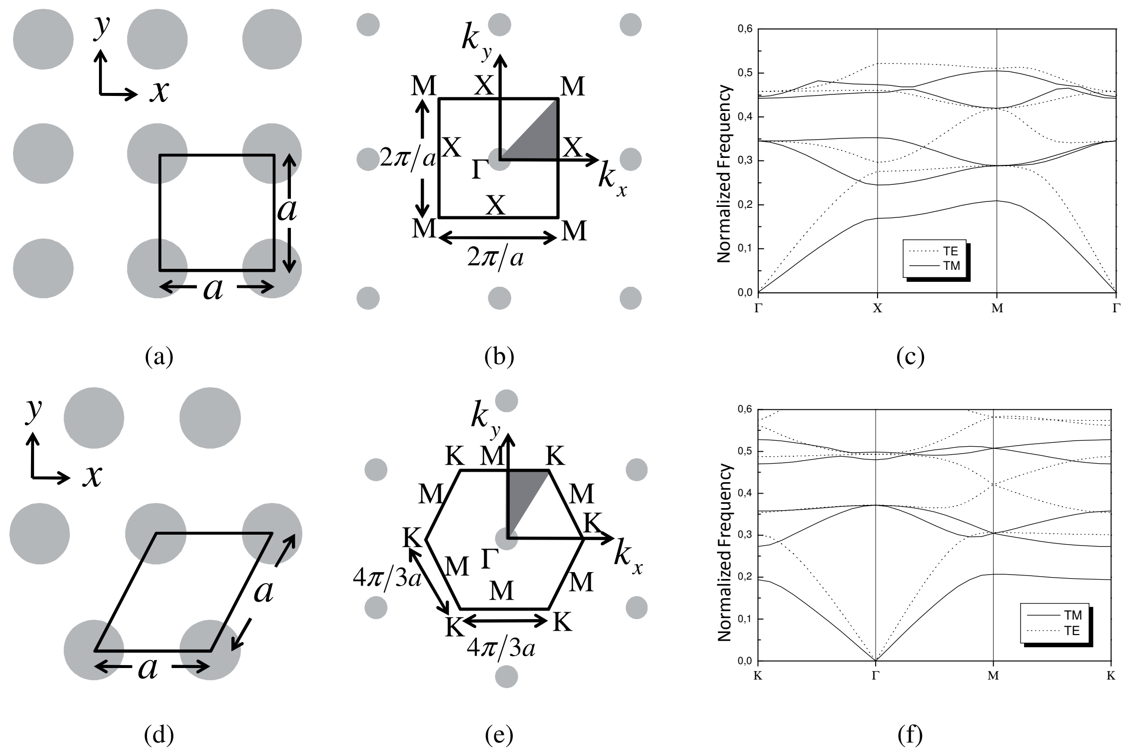

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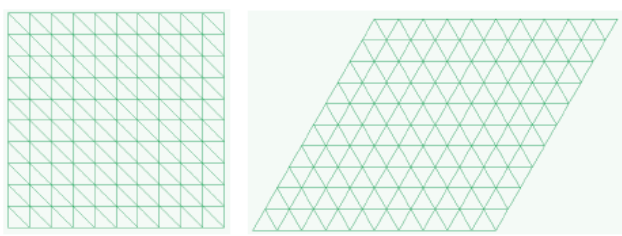
Figure 1. (a) square lattice and its unitary cell. (b) Brillouin region. (c) band structure for dielectric rods with n=3.6 in air with r/*a*=0.35. (d) Triangular lattice and its unitary cell. (e) Brillouin region (f) band structure for dielectric rods with n=3.6 in air with r/*a*=0.33**.**

In the present case, an eigenvalues problem needs to be solved in order to obtain the forbidden band gaps. Details of the algorithm can be found in [Malheiros, 2011].

We considered a crystal composed by silicon and air whose refractive indexes are n=3.476 and n=1.0, respectively. The used meshes are shown in Figure 2. Observe the division of the geometries in small triangles. There are 200 triangles in each mesh; each triangle can be composed either by silicon or by air, resulting in a chromosome with 200 genes. The problem to be solved can be simplified as follows: what material should be put in each triangle in order to maximize the forbidden band gap? Of course, we have 2^{200} possibilities.



(a) (b)

Figure 2. Unitary cell of the (a) square and (b) triangular lattices, respectively.

3. Genetic Algorithm

For our optimization problem, it is almost impossible to evaluate the 2^{200} possible solutions because it would be cumbersome and also time consuming problem. In this way the genetic algorithms has been chosen for the optimization because it has been previously used for solving electromagnetic problems. The considered genetic algorithm has the following stages:

a) Initialization: The initial population is generated and each individual is represented as a binary sequence. If a gene is "1", the respective triangle is composed by silicon, otherwise air is considered is such triangle. We have performed simulations with 20 and 50 individuals.

b) Evaluation: The fitness of each individual is evaluated by the objective function given by:

$$Objective\ Function = \frac{Frequency_{top} - Frequency_{bottom}}{Frequency_{middle}}$$

c) Selection: Individuals are selected for reproduction and their probability to be selected is proportional to its fitness, the Roulette Wheel has been considered here.

d) Crossover: Two schemes have been considered in this work. The two-points and the uniform crossover. In the former, two position of the chromosome are randomly chosen and the genes contained these points are exchanged between two individuals. In the later, a binary mask is randomly generated and the value "1" or "0" indicate which father will contribute with its gene at that position. This process is shown in Figure 3, for this example the crossover using the binary mask [0,0,1,0,0,1,1,1,0,0,1,0,1,1,0,1,0,0,1,0] is presented. It is

expected a better performance for the uniform crossover [Linden 2008].
e) Mutation: It has been adopted a mutation every 5 generations
f) Elitism: generated children/offspring are considered as parents in next generation preserving the best individual.

g) Ending: We consider 1000 generations as stop criterion in all cases.
The genetic algorithm has been implemented by using Matlab® following the flowchart shown in Fig. 4.

4. Photonic Crystal Band Gap Optimization

The forbidden band gap of square and triangular lattices have been optimized and in all cases populations of 20 or 50 individuals have been considered. The materials considered in each triangle of the mesh can result in elements with triangular or parallelogram shape. A parallelogram consists of two adjacent triangles. The resulting optimized structure is shown in Figure 5 and Figure 6 with seven unitary cells in the horizontal and vertical directions.

In both cases, the final structures are very similar. The band gap results are summarized in Table 2, where it can be verified a better performance of the uniform crossover over the two-points one. It can be explained as follows: The uniform crossover enhances the search universe avoiding local optimal values because the children generated by it have higher diversity when compared with the children generated by the two-point scheme. A better performance can be observed for the forbidden band gap of the triangular lattice.

The evolution of the best individuals for each case (lattice, type of element, crossover) is shown in Figure 7 and Figure 8. In all cases, the faster convergence is observed for the uniform crossover. The mean time processing for each case is shown in Table 2. An Intel® Celeron® CPU 430 @ 1.80GHz with 2.0 Gb de RAM has been used in all simulations.

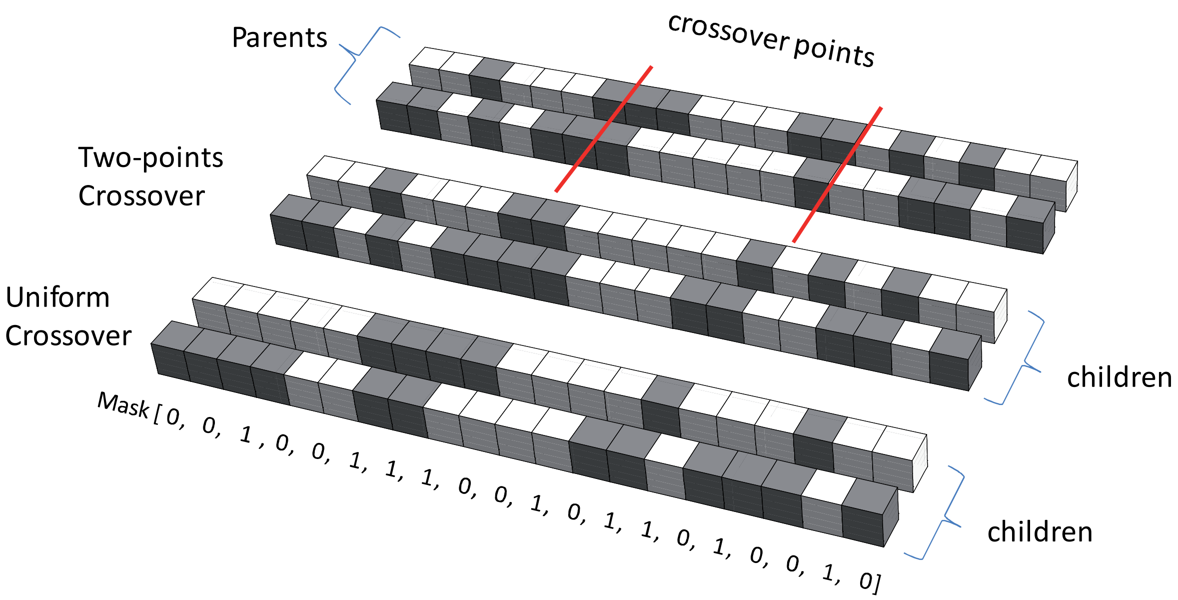


Figure 3. Two-points and uniform crossover.

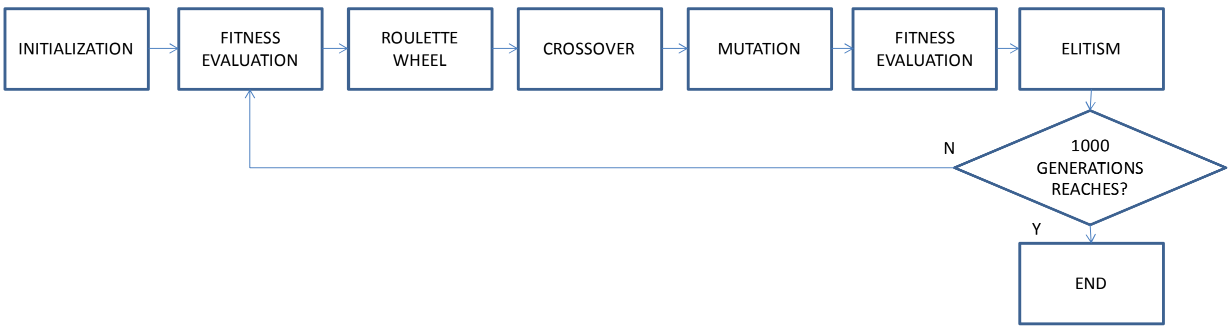
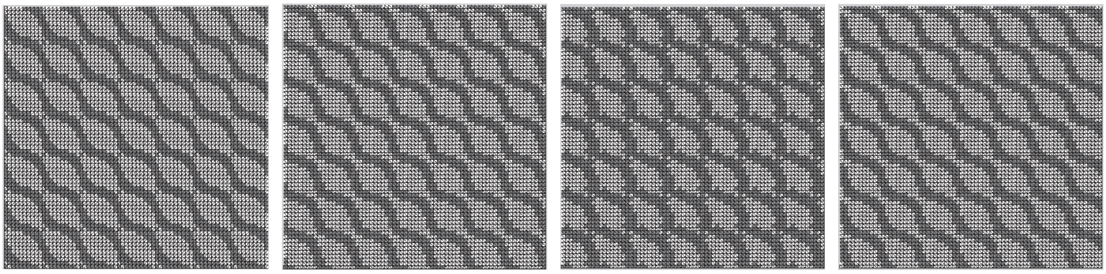
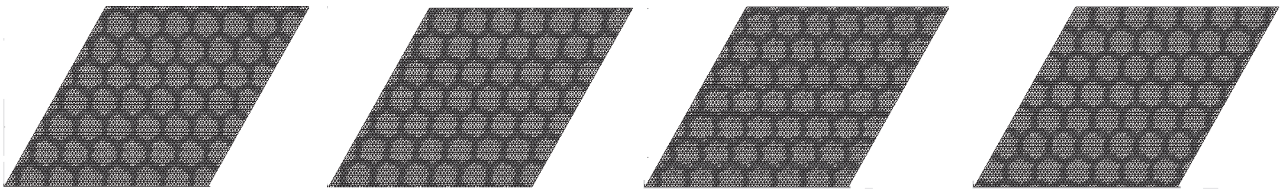


Figure 4. Flowchart of the Genetic Algorithm.



(a) (b) (c) (d)

Figure 5. Optimized photonic crystals for TM_{12} modes for the square lattice considering parallelogram elements (a) two-points crossover and 20 individuals (b) uniform crossover and 20 individuals (c) two-points crossover and 50 individuals (d) uniform crossover and 50 individuals.

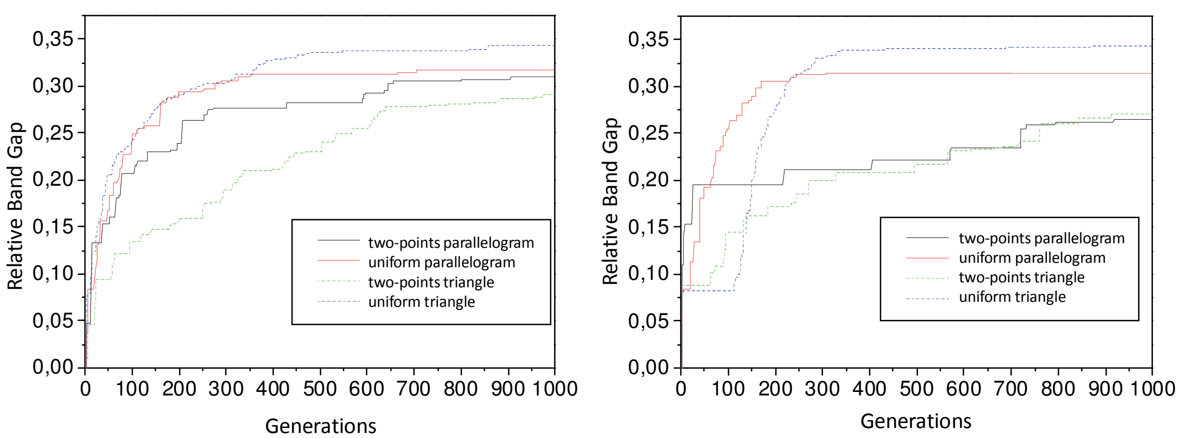


(a) (b) (c) (d)

Figure 6. Optimized photonic crystals for TM_{12} modes for the triangular lattice considering parallelogram elements (a) two-points crossover and 20 individuals (b) uniform crossover and 20 individuals (c) two-points crossover and 50 individuals (d) uniform crossover and 50 individuals.

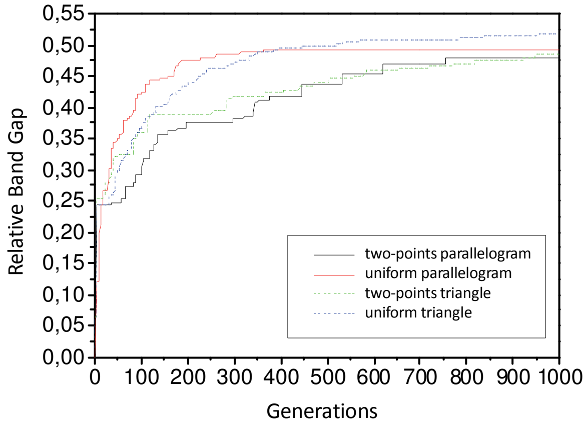
Table 1. Fitness of the Forbidden Band Gap TM_{12}

Square Lattice Crystals							
20 individuals				50 individuals			
Parallelogram Element		Triangular Element		Parallelogram Element		Triangular Element	
Two-points	Uniform	Two-points	Uniform	Two-points	Uniform	Two-points	Uniform
30.95%	31.67%	29.20%	34.32%	26.49%	31.37%	27.06%	34.25%
Triangular Lattice Crystals							
20 individuals				50 individuals			
Parallelogram Element		Triangular Element		Parallelogram Element		Triangular Element	
Two-points	Uniform	Two-points	Uniform	Two-points	Uniform	Two-points	Uniform
48.09%	49.18%	48.51%	51.74%	44.81%	49.20%	42.27%	51.65%

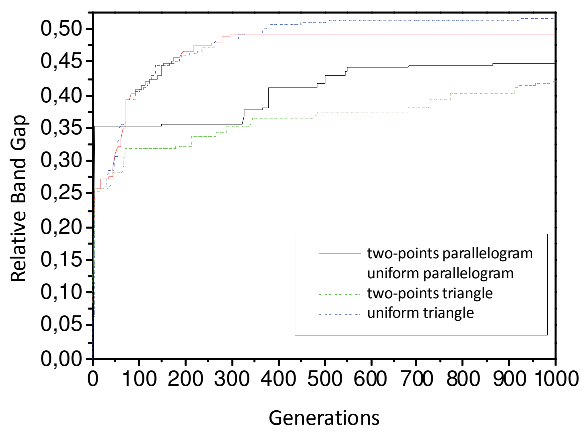


(a) (b)

Figure 7. Evolution of the fitness of the best individuals for the square lattice with population of (a) 20 individuals and (b) 50 individuals.



(a)



(b)

Figure 8. Evolution of the fitness of the best individuals for the triangular lattice with population of (a) 20 individuals and (b) 50 individuals.

Table 2. Mean processing time for each generation evaluation.

	Square Lattice Crystal			
	Two-points Crossover		Uniform Crossover	
	20 individuals	50 individuals	20 individuals	50 individuals
Parallelogram Element	1min 42s	3min 36s	1min 30s	3min 24s
Triangular Element	1min 12s	3min 24s	1min 18s	3min 12s
	Triangular Lattice Crystal			
	Two-points Crossover		Uniform Crossover	
	20 individuals	50 individuals	20 individuals	50 individuals
Parallelogram Element	2min	5min	2min 18s	5min 18s
Triangular Element	2min	4min 56s	2min	5min

5. Conclusions

As a conclusion, the performance of the uniform crossover was better than the two-points in all the cases of forbidden band gap optimization of two dimensional photonic crystals related to the TM_{12} band gap. The uniform crossover enhances the search universe avoiding local optimal values because the children generated by it have higher diversity when compared with the children generated by the two-point scheme. A better performance can be even observed for the triangular lattice case.

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